Module 2 – Linear Fiber Propagation

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Introduction

The goal of modules 2-5 is to build an understanding of nonlinear fiber optics and in particular temporal optical solitons. To accomplish this goal we shall refer to material from module 1 of the graduate super-course, *Electromagnetic Wave Propagation*, as well as pointing to the connections with several undergraduate (UG) modules as we go along. We start in module 2 with a treatment of linear pulse propagation in fibers that will facilitate the transition to nonlinear propagation and also establish notation, and in module 3 we shall discuss numerical pulse propagation both as a simulation tool and an aid to understanding. Nonlinear fiber properties will be discussed in module 4, and temporal solitons arising from the combination of linear and nonlinear properties in optical fibers shall be treated in module 5.

The learning outcomes for this module include

- The student will be conversant with the LP modes of optical fibers and how to calculate the properties of the fundamental mode.
- The student will be able to assess the dispersion properties of optical fibers including the group velocity and the GVD parameter.
- The student will be able to derive the slowly varying envelope equation for propagation in fibers and be conversant with the physical content of the terms involved.
- The student will be able to calculate the evolution of a Gaussian pulse in an optical fiber and assess the dispersion-induced pulse broadening involved.

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- The student will be conversant with the idea of pulse chirp and how it relates to the idea of the instantaneous frequency shift.

2.1 Linearly polarized (LP) fiber modes

For a fiber with propagation axis along the z-axis the modes are generally classified as ***TE, TM,*** or ***Hybrid*** depending on the state of polarization of the electric and magnetic fields with respect to the Cartesian axes. However, it is often the case that for telecommunications applications the modes may be approximated as linearly polarized yielding *LP modes* which are linearly polarized orthogonal to the z-axis. In particular, this is the case in the so-called *weak guidance approximation* in which the refractive index n_1 of the core is greater than but close to the refractive index n_2 of the cladding, see **Figure. 2.1** below

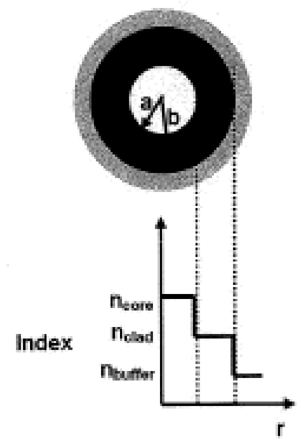

Figure. 2.1 *Step-index fiber geometry*.

Physically the weak guidance approximation means that the fundamental fiber model will not be tightly confined to the fiber core, but the mode field will also protrude into the cladding. For telecommunication applications the normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 > 0$ is typically 0.02 or less in which case the weak guidance approximation is valid. The weak guidance condition is desirable as it is consistent with the condition for single-mode operation of a step-index fiber $V < 2.405$, where the V-parameter is $V = ka\sqrt{n_1^2 - n_2^2}$, a being the fiber core radius, $k = \omega / c = 2\pi / \lambda$ the free-space wave vector magnitude, λ the wavelength, and ω the field frequency. Clearly a smaller value of Δ is conducive to single-mode operation.

Hereafter we shall assume that the fiber under consideration is single-mode, $V < 2.405$, and that the weak guidance approximation is valid. In this case the lowest-order or fundamental mode at frequency ω is designated as the LP_{01} mode (it corresponds to the HE_{11} mode from the exact treatment), it has a transverse mode profile $F(x, y, \omega)$ in the x-y plane that roughly has a Gaussian shape, and is linearly polarized perpendicular to the z-axis, (e.g., $\mathbf{e} = \mathbf{x}$ or $\mathbf{e} = \mathbf{y}$),

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where $\mathbf{e}$ is the unit polarization vector for the fundamental mode, and $\mathbf{x},\mathbf{y}$ are the unit vectors along the x and y axes.

To obtain an equation for the fundamental LP_{01} mode we start from the wave equation

$$\left(\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} ,$$

(Equation 2.1)

and for a modal solution at frequency ω we write the vector electric field in complex representation as

$$\mathbf{E}(\mathbf{r},t) = \mathbf{e}A(\omega)F(x,y,\omega)e^{i(\beta(\omega)z-\omega t)} .$$

(Equation 2.2)

Here $\beta(\omega)$ is the frequency-dependent *mode propagation constant*, and the linear optical polarization due to the fiber is given by

$$\mathbf{P}(\mathbf{r},t) = \epsilon_0 \chi^{(1)}(x,y)\mathbf{E}(\mathbf{r},t) ,$$

(Equation 2.3)

where the spatially dependent *linear susceptibility* $\chi^{(1)}(x,y)$ reflects the transverse variation of the refractive index profile of the fiber, see **Figure. 2.1** for the example of a cylindrically symmetric step index fiber, the refractive index being given by $n^2 = 1 + \chi^{(1)}$. Combining **Equation. (2.1)-(2.3)** we obtain the equation for the fundamental mode

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2 n^2(x,y,\omega)\right)F(x,y,\omega) = \beta^2(\omega)F(x,y,\omega) .$$

(Equation 2.4)

This equation has the form of an eigenvalue problem with $\beta^2(\omega)$ playing the role of the eigenvalue and $F(x,y,\omega)$ the eigenmode.

For a step-index fiber in the weak guidance approximation the fundamental mode profile may be approximated as a Gaussian to reasonable accuracy

$$F(x,y,\omega) \propto e^{-(x^2+y^2)/w^2} .$$

(Equation 2.5)

Figure 2.2 shows the variation of the normalized best-fit Gaussian spot size (w/a) versus V

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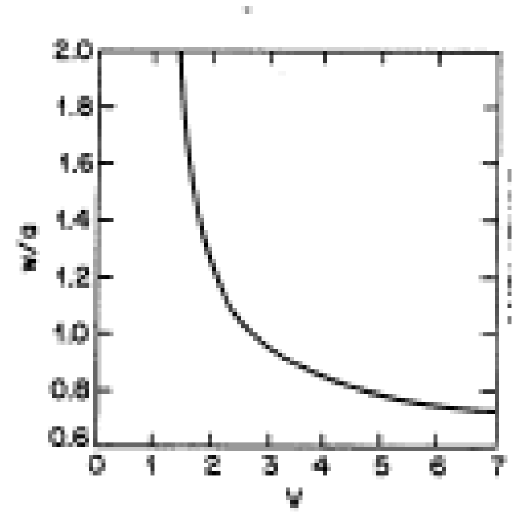

Figure. 2.2 *Normalized best-fit Gaussian spot size* (w/a) *as a function of the V-parameter*.

and **Figure. 2.3** shows the corresponding normalized propagation constant

$$b = \frac{[(\beta / k) - n_2]}{(n_1 - n_2)} ,$$

(Equation 2.6)

as a function of the V-parameter for a variety of LP modes, the fundamental mode corresponding to the LP $_{01}$ mode.

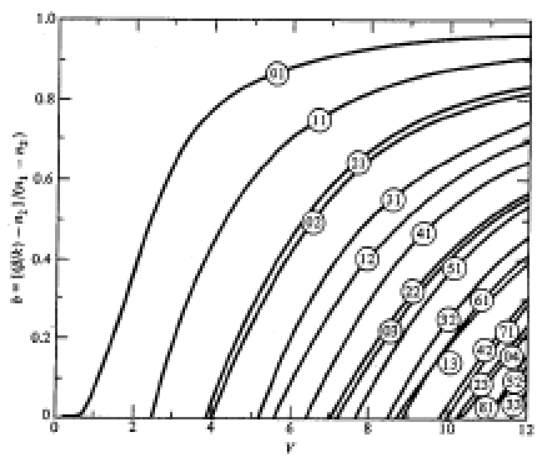

Figure. 2.3 *Normalized propagation parameter b versus V-parameter for various* LP_{jm} *modes with the jm values shown in the circles. The fundamental mode corresponds to the* LP_{01} *mode*.

The results in **Figures. 2.2 and 2.3** allow one to calculate the fundamental mode profile and the mode propagation constant for a step-index fiber for fixed given values of the V-parameter, fiber parameters, and frequency or wavelength. For more general fiber structures, eg. photonic crystal fibers, the fundamental mode can be solved numerically. For the remainder of this development

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we assume that both the fundamental mode profile $F(x, y, \omega)$ and the corresponding mode propagation constant $\beta(\omega)$ are known for a given fiber geometry.

For additional reading and details on the material covered in this section see Chap. 2.4 of Ref. [1] and Chap. 2.2 of Ref. [2].

2.2 Chromatic dispersion and fiber losses

In general both the fundamental mode profile $F(x, y, \omega)$ and the corresponding mode propagation constant $\beta(\omega)$ vary with frequency ω. This is relevant since pulses propagating in the fiber can be viewed as a superposition of many or a range of frequencies. For our purposes we shall assume that all pulses of interest have a center frequency ω_0 and a spread of frequencies $\Delta\omega << \omega_0$. With this caveat we may safely neglect the frequency variation of the fundamental mode profile and replace $F(x, y, \omega_0) \equiv F(x, y)$.

On the other hand it is of paramount importance to take account of the frequency variation of the mode propagation constant $\beta(\omega)$ as this incorporates *chromatic dispersion* within the fiber whereby different frequencies experience different propagation constants for the same fundamental mode. Chromatic dispersion is introduced in UG module 5, and underlies all of the linear pulse propagation properties that we shall discuss. There are two sources of chromatic dispersion, namely *material dispersion* arising from the fact that the core and cladding refractive-indices $n_{1,2}(\omega)$ both depend on frequency, and *waveguide dispersion* arising from wave propagation in the fiber structure that would be present even in the absence of material dispersion.

So far we have assumed that the refractive indices of the materials comprising the fiber are real-valued thereby neglecting any losses. In the wavelength region of interest to telecommunications fiber losses are dominantly due to material losses and Rayleigh scattering, and these will be different for the different media comprising the fiber and will vary with frequency. The optical properties of the fiber materials are discussed at length in UG module 2. Analogous to the refractive index profile $n(x, y, \omega)$ for the fiber there will in general also be an absorption profile $\alpha(x, y, \omega)$ reflecting the composition of the fiber. Then based on the notion that all pulses of interest have a center frequency ω_0 and a spread of frequencies $\Delta\omega << \omega_0$, and that the fundamental mode profile may be approximated as $F(x, y) \equiv F(x, y, \omega_0)$, the net absorption experienced by the propagating mode may be approximated as

$$\alpha(\omega_0) \approx \frac{\int dx \int dy \alpha(x, y, \omega_0) |F(x, y)|^2}{\int dx \int dy |F(x, y)|^2},$$

(Equation 2.7)

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which represents a spatially averaged value of the absorption experienced by the fundamental fiber mode.

Figure 2.4 shows the variation of the fiber loss $\alpha_{dB} = 4.343\alpha$ in dB versus center wavelength λ for a single-mode silica fiber with a minimum loss around 1.55 μm.

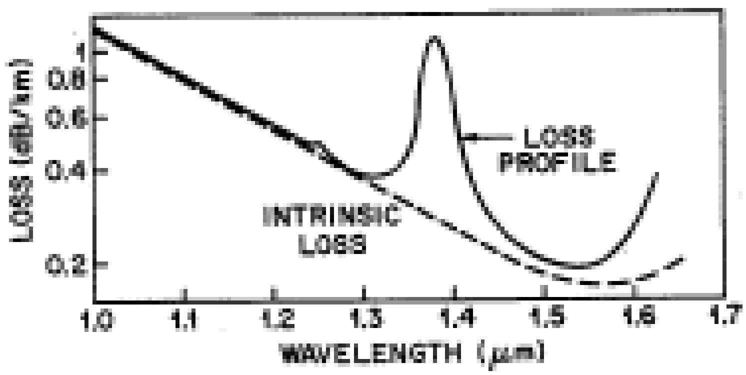

Figure. 2.4 *Loss in dB versus wavelength for a single-mode silica optical fiber. The dashed line shows the intrinsic Rayleigh scattering loss.*

Such fibers with very low losses in the 1.3-1.6 μm range are employed for fiber-optic telecommunications. Attenuation or loss in optical fibers is discussed in detail in UG module 3. Hereafter we shall assign a value to the fiber absorption coefficient α in which case the amplitude $A(\omega)$ of the electric field appearing in **Equation. 2.2** will vary according to Beer's law

$$A(\omega, z) = A(\omega, 0)e^{-\alpha z/2},$$

(Equation 2.8)

with $A(\omega, 0)$ the amplitude at the fiber input $z = 0$.

For additional reading and details on the material covered in this section see Chap. 3.1 of Ref. [1] and Chap. 1.2 of Ref. [2].

2.3 Group velocity and group velocity dispersion

The mode propagation constant is often written in the form $\beta(\omega) = \omega n(\omega)/c$ with $n(\omega)$ the frequency-dependent refractive index for the mode. For a field frequency ω propagating in the fiber the phase-velocity is $v_p(\omega) = \omega/\beta(\omega) = c/n(\omega)$. The phase velocity describes the motion of phase fronts for a carrier plane-wave at ω. In contrast, for a pulse of center frequency ω the group velocity that dictates the velocity of the pulse envelope that multiplies the carrier is given by

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$$v_g(\omega) = \frac{\partial \omega}{\partial \beta} = \frac{c}{n_g(\omega)} ,$$

(Equation 2.9)

where the group index is given by $n_g(\omega) = \left(n + \omega \frac{dn}{d\omega} \right)$. The phase and group velocities are key concepts for pulse propagation in fibers and are discussed in UG module 4.

As stated earlier we are interested in pulses with center frequency ω_0 and a spread of frequencies $\Delta\omega << \omega_0$. Based on this it is useful to Taylor expand $\beta(\omega)$ around the center frequency to give

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2!}(\omega - \omega_0)^2 + \frac{\beta_3}{3!}(\omega - \omega_0)^3 + \ldots,$$

(Equation 2.10)

Where the real coefficients β_m are given by

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} .$$

(Equation 2.11)

The coefficient β_0 is clearly the mode propagation constant at the carrier or central frequency $\omega = \omega_0$. Using **Equation. 2.10** we then we find that the group velocity at the carrier frequency is $v_g = v_g(\omega_0) = \partial\omega / \partial\beta = 1/\beta_1$, that is we identify the coefficient $\beta_1 = 1/v_g$ as the inverse group velocity. If we retain the first three terms shown on the right-hand-side of **Equation. 2.10** we find the frequency-dependent group velocity

$$v_g(\omega) = \frac{v_g}{1 + \beta_2(\omega - \omega_0)v_g + \beta_3(\omega - \omega_0)^2 v_g / 2} .$$

(Equation 2.12)

From this equation we see that the terms involving the coefficients $\beta_m, m > 1$ produce a frequency dependence of the group velocity with respect to the central frequency, and these terms give rise *to group velocity dispersion* (GVD). The coefficient β_2 is called the GVD parameter, and the coefficients $\beta_m, m > 2$ give rise to *higher-order GVD.*

For our treatment we shall consider only the terms with coefficients $\beta_0, \beta_1, \beta_2$ in **Equation. 2.10** giving

$$\beta(\omega) \approx \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2.$$

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(Equation 2.13)

Then note that according to **Equation. 2.12** the group velocity decreases with increasing frequency for the case of normal dispersion $\beta_2 > 0$, but increases with increasing frequency for the case of anomalous dispersion $\beta_2 < 0$. **Figure 2.5** shows the GVD parameter β_2 versus central wavelength for a single mode fiber, and we see that the GVD changes from normal to anomalous for wavelengths greater than 1.27 μm. We shall consider the case of anomalous dispersion when we study solitons in optical fibers.

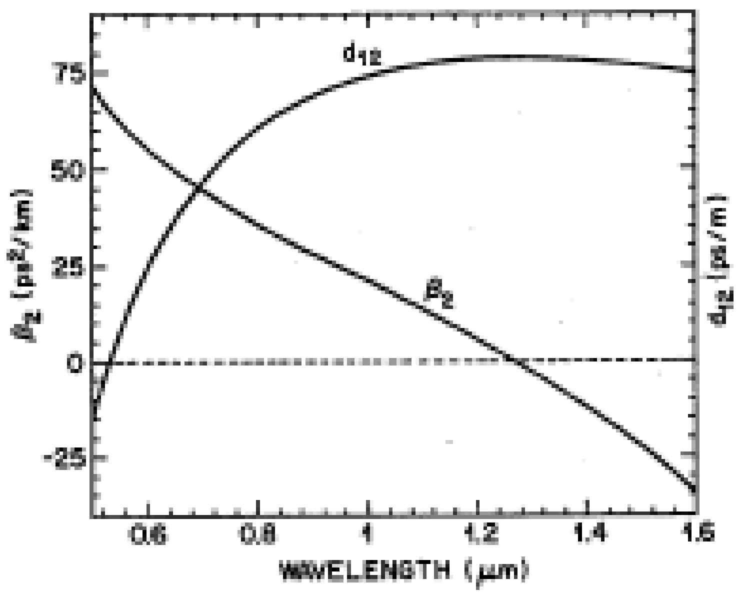

Figure. 2.5 Variation *of the dispersion parameter* β_2 *with wavelength for fused silica*.

For additional reading and details on the material covered in this section see Chapter. 3.2 and Appendix F of Ref. [1] and Chap. 1.2 of Ref. [2].

2.4 Pulse propagation in fibers

We would now like to apply the ideas of the previous section to pulse propagation in a fiber. To do this we follow the same approach as UG module 4 in which a pulse was constructed from a superposition of monochromatic mode solutions. In particular, we would like to find a general solution for a pulse of central frequency ω_0 propagating in a single mode fiber. What we have discussed so far is a single frequency or monochromatic field propagating in the fiber, the vector electric field taking the form shown in **Equation. (2.2)**

$$\mathbf{E}(\mathbf{r},t) = \mathbf{e}A(\omega)F(x,y,\omega)e^{i(\beta(\omega)z-\omega t)} ,$$

(Equation 2.14)

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this solution obeying Maxwell's equations in the fiber. Since we are currently discussing linear propagation a general pulsed solution may be written as a summation of such monochromatic solutions that we write in integral form

$$\mathbf{E}(\mathbf{r},t) = \int d\omega \mathbf{e} A(\omega) F(x,y,\omega) e^{i(\beta(\omega)z - \omega t)} \ ,$$

(Equation 2.15)

where $A(\omega)$ controls the weighting of each monochromatic field component in the pulse. So far the solution is exact. But recall that we neglect any variation in the fundamental mode profile with frequency $F(x,y,\omega) \approx F(x,y,\omega_0) \equiv F(x,y)$. Furthermore, substituting **Equation. 2.13** for $\beta(\omega)$ in **Equation. (2.15)** and setting $\omega = \omega_0 + \Delta\omega$ we obtain

$$\mathbf{E}(\mathbf{r},t) = \mathbf{e} F(x,y) e^{i(\beta_0 z - \omega_0 t)} A(z,t) \ ,$$

(Equation 2.16)

where

$$A(z,t) = \int d(\Delta\omega) A(\Delta\omega) e^{i(\beta_1 \Delta\omega z - \beta_2 \Delta\omega^2 z/2 - \Delta\omega t)} \ .$$

(Equation 2.17)

The structure of the pulsed solution in **Equation. 2.16** is key: First the pulse is linearly polarized with unit polarization vector $\mathbf{e}$, and it has the fundamental mode profile $F(x,y)$. Second, the factor $e^{i(\beta_0 z - \omega_0 t)}$ acts as a carrier plane-wave at the carrier frequency ω_0 whose phase fronts propagate at the phase velocity v_p. Finally, we have the pulse envelope $A(z,t)$ that describes the temporal dynamics of the pulse as a function of propagation distance z along the fiber.

2.5 Slowly varying envelope approximation

Hereafter we shall focus our attention on the field envelope $A(z,t)$ and our next task is to find an equation of motion for the field envelope. We anticipate that the equation of motion for the field envelope will involve the partial derivatives of $A(z,t)$ with respect to z and t, so we form these derivatives based on **Equation. 2.17**

$$\frac{\partial A}{\partial z} = \int d(\Delta\omega) A(\Delta\omega) \underbrace{[i\beta_1 \Delta\omega - i\beta_2 \Delta\omega^2 / 2]} e^{i(\beta_1 \Delta\omega z - \beta_2 \Delta\omega^2 z/2 - \Delta\omega t)} \ ,$$

(Equation 2.18)

$$\frac{\partial A}{\partial t} = \int d(\Delta\omega) A(\Delta\omega) \underbrace{[-i\Delta\omega]} e^{i(\beta_1 \Delta\omega z - \beta_2 \Delta\omega^2 z/2 - \Delta\omega t)} \ ,$$

(Equation 2.19)

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$$\frac{\partial^2 A}{\partial t^2} = \int d(\Delta\omega) A(\Delta\omega) \underbrace{[-\Delta\omega^2]} e^{i(\beta_1 \Delta\omega z - \beta_2 \Delta\omega^2 z/2 - \Delta\omega t)} \ .$$

(Equation 2.20)

To proceed we notice that these three expressions differ only in the underbraced terms appearing in the integrands, so that if we form a linear combination of these three expressions such that the combination of the underbraced terms cancel for each $\Delta\omega$ we will have a suitable equation of motion. Using this approach you will verify in the homework that the equation of motion for the field envelope is

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = 0 \ ,$$

(Equation 2.21)

this being the so-called *slowly varying envelope equation* for pulse propagation in a fiber. It is based on the approximation that the field envelope $A(z,t)$ varies slowly in time and space in comparison to the plane-wave carrier. Equation 2.21 is the foundation for our discussion of linear pulse propagation in fibers. Here we have developed the envelope equation by starting from the exact solution in **Equation. 2.17**. One may also derive this equation from Maxwell's equations as is discussed in Section 2.3 of Ref. [2].

In order to gain some insight into and experience with **Equation. 2.21** we first consider the case with no GVD and set $\beta_1 = 1/v_g, \beta_2 = 0$ in Eq. 2.21 to obtain

$$\frac{\partial A}{\partial z} + \frac{1}{v_g} \frac{\partial A}{\partial t} = 0 \ .$$

(Equation 2.22)

You may check that this equation has traveling wave solutions of the form $A(z,t) = A_0(t - z/v_g)$ which represent pulses with temporal profile $A_0(t)$ at $z = 0$ that propagate undistorted along the z-axis at the group velocity v_g with respect to a stationary lab frame. **Equation. 2.21** conforms to our notion that in the absence of GVD pulses should propagate at the group velocity. It may be tempting to always choose an operating condition such that $\beta_2 = 0$ so that **Equation. 2.21** reduces to **(2.22)**, this corresponding to the zero-GVD operating point. However, in this case we could no longer ignore the higher-order dispersion terms involving the coefficients $\beta_m, m > 2$, and these lead to an even more complicated envelope equation involving higher-order time derivatives. For this reason we shall always assume that β_2 is always the dominant GVD term and that higher-order dispersion may be neglected.

As a second example we consider a continuous wave (CW) field but include the effect of absorption. For this case we set the time derivatives in **Equation. 2.21** to zero to obtain

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$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A \,,$$

(Equation 2.23)

which has the solution $A(z,T) = A(0,T)e^{-\alpha z/2}$ that agrees with **Equation. 2.8** and Beer's law.

As a final example we consider a transformation to a reference frame, the so-called retarded frame, moving at the group velocity to simplify **Equation. 2.21**. In particular, we change to new coordinates z and $T = (t - z/v_g) = (t - \beta_1 z)$. In the moving reference frame **Equation. 2.21** becomes

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} = -\frac{\alpha}{2} A(z,T)$$

(Equation 2.24)

including absorption. In the absence of GVD and absorption the solution to this equation is $A(z,T) = A_0(T)$, that is the pulse will appear stationary with initial profile $A_0(T)$ since we are in the retarded frame moving at the group velocity. This is in contrast to the solution $A(z,t) = A_0(t - z/v_g)$ in the lab frame in which the pulse is moving. The distinction between the lab and retarded frames is illustrated in **Figure. 2.6**. It is important to understand that the time variable $T = (t - z/v_g)$ is normally taken as time relative to the pulse center. To see this realize that for a fixed value of T we must follow the characteristic $t = (T + z/v_g)$, so T parameterizes time slices in the pulse at $z = 0$. Hereafter we shall use Eq. 2.24 in the moving reference frame.

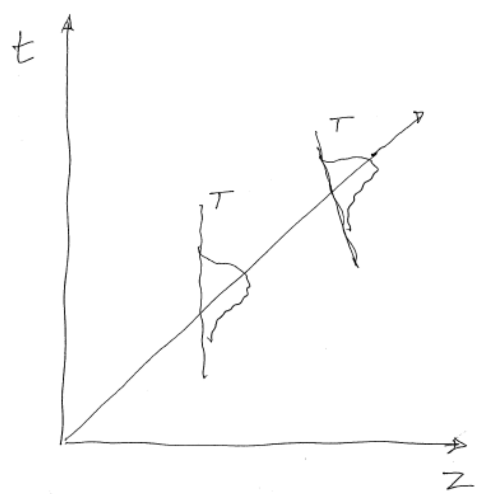

Figure. 2.6 *This Figureure is intended to illustrate the difference between the lab frame coordinates (z,t) and the retarded coordinates (z,T).*

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So far we have said nothing about the dimensions of the field envelope. Here we shall adopt a scaling such that $|A(z,T)|^2$ represents the pulse power in the time slice labeled by T and for a propagation distance z. For an incident pulse of peak power P_0 it will prove useful to introduce the scaled pulse envelope defined by

$$A(z,T) = \sqrt{P_0}U(z,T) \ ,$$

(Equation 2.25)

so that $U(z,T)$ is dimensionless, and the slowly-varying envelope equation becomes

$$\frac{\partial U}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 U}{\partial T^2} - \frac{\alpha}{2}U(z,T) \ .$$

(Equation 2.26)

We shall employ both **Equations. 2.24 and 2.26** in our discussion.

For additional reading on the material covered in this section see Chapter. 2.3 of Ref. [2].

2.6 Gaussian pulse propagation with GVD and loss

We next turn to a very important exact solution of **Equation. 2.26** in the presence of GVD and loss, namely a Gaussian pulse solution. The Gaussian solution is also described in UG module 4, but not from the perspective of the slowly varying envelope equation. Rather than getting bogged down in the derivation of this solution here I quote the solution and you shall verify it in the homework. In particular we consider an initial Gaussian pulse of the form $U(0,T) = \exp(-T^2/2T_0^2)$ at $z=0$ with the real parameter T_0 the input pulse width. Then for z>0 Eq. 2.26 dictates that the Gaussian pulse evolves according to

$$U(z,T) = \frac{T_0 e^{-\alpha z/2}}{(T_0^2 - i\beta_2 z)^{1/2}} \exp\left(-\frac{T^2}{2(T_0^2 - i\beta_2 z)}\right).$$

(Equation 2.27)

To gain insight into this solution it is useful to re-express it in the form

$$U(z,T) = U(z,0)\exp\left(-\frac{T^2}{2T_1^2(z)} + i\phi(z,T)\right),$$

(Equation 2.28)

where $U(z,0)$ is the value of the envelope at pulse center, the real parameter $T_1(z)$ is the pulse width as a function of propagation distance z, and $\phi(z,T)$ is a time-dependent phase factor. In particular, by comparing **Equations. 2.27 and 2.28** we find that the pulse width evolves according to

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$$T_1(z) = T_0\left[1+\left(\frac{z}{L_D}\right)^2\right]^{1/2},$$

(Equation 2.29)

where $L_D = T_0^2/|\beta_2|$ is the *dispersion length*. Then, for example, for a propagation distance $z = L_D$ the initial pulse of width T_0 broadens to $T_1(L_D) = \sqrt{2}T_0$, for $z = 2L_D$ to $T_1(L_D) = \sqrt{5}T_0$, and so on. We also find that as the pulse broadens the magnitude of its peak decreases

$$|U(z,0)| = \frac{T_0 e^{-\alpha z/2}}{(T_0^4 + |\beta_2|^2 z^2)^{1/4}}.$$

(Equation 2.30)

Figure 2.7 shows an example of dispersion-induced pulse broadening for propagation distances $(z/L_D) = 0, 2, 4$ and illustrates the increase in pulse width and decrease in peak power with propagation distance. In this example the linear absorption is absent (i.e., $\alpha = 0$).

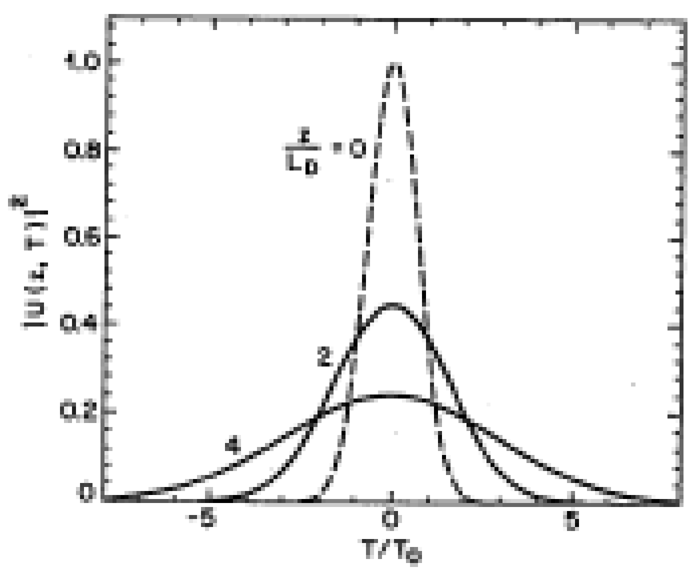

Figure. 2.7 *This Figure shows dispersion-induced broadening of an input Gaussian pulse for propagation distances* $z = 0, 2L_D, 4L_D$.

We learn two very important things from the Gaussian solution. First GVD gives rise to *dispersion-induced pulse broadening* whereby an initial pulse broadens upon propagation through the fiber. Second, the length scale over which the broadening occurs is the dispersion length

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$$L_D = \frac{T_0^2}{|\beta_2|}.$$

(Equation 2.31)

Thus, for a given input pulse duration T_0 pulse broadening occurs on a shorter length scale L_D for larger GVD parameter β_2 , and for a fixed GVD parameter pulse broadening occurs on a shorter length scale L_D for shorter input pulses. The dispersion length L_D is a very important length scale for optical fiber propagation as it determines the length scale over which pulse broadening occurs.

For additional reading on the material covered in this section see Chapter. 3.2 of Ref. [2].

2.7 Pulse broadening and chirp

The solution for the pulse width $T_1(z)$ in **Equation. 2.29** indicates that the pulse broadening only depends on the magnitude of the GVD parameter β_2 not on its sign, and therefore an initial Gaussian pulse will always broaden if the GVD is non-zero. This is due to the special nature of the initial Gaussian pulse $U(0,T) = \exp(-T^2 / 2T_0^2)$ that is modulated in amplitude but not phase. Next we consider a chirped Gaussian pulse of the form

$$U(0,T) = \exp\left(-\frac{(1+iC)}{2}\frac{T^2}{T_0^2}\right) = \exp\left(-\frac{T^2}{2T_0^2} + i\varphi(T)\right),$$

(Equation 2.32)

where C is the chirp parameter. The effect of pulse chirp on Gaussian pulse propagation is also described in UG module 4. We state without proof that in this case the pulse width evolves according to the expression

$$\frac{T_1(z)}{T_0} = \left[\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{z}{L_D}\right)^2\right]^{1/2},$$

(Equation 2.33)

which reduces to **Equation. 2.29** if $C = 0$. An interesting feature of this solution is that when the sign of the product $C\beta_2$ is negative the pulse width may actually initially decrease with increasing propagation distance. An example of this is shown in **Figure. 2.8** which shows the broadening factor (T_1 / T_0) versus scaled propagation distance (z / L_D) for chirp parameters $C = -2, 0, 2$ for $\beta_2 > 0$, and we see the pulse initially narrowing for $C = -2$.

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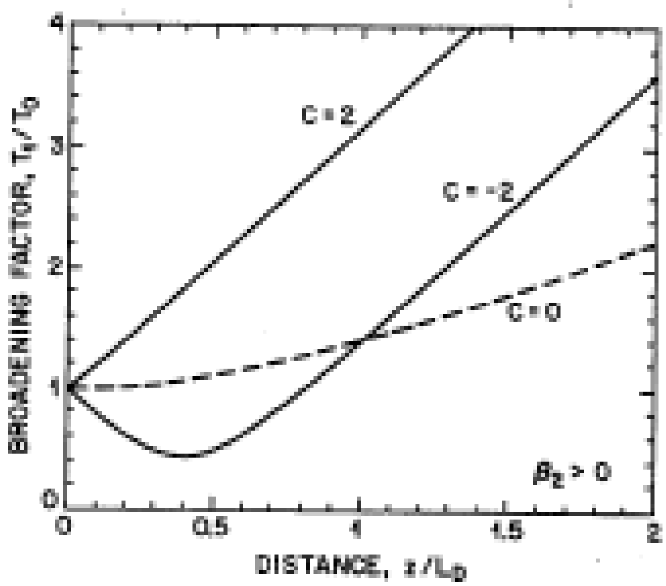

Figure. 2.8 *This Figure shows the broadening factor versus propagation distance for chirp parameters* $C = -2, 0, 2$.

We would like to obtain an intuitive picture of how chirp affects the pulse propagation. To do this we develop the approximate notion of the instantaneous frequency shift. In particular, for an initial chirped pulse with phase variation $\phi(T)$ the instantaneous frequency shift with respect to the carrier frequency ω_0 is defined as

$$\delta\omega = -\frac{\partial\phi(T)}{\partial T}.$$

(Equation 2.34)

For the chirped Gaussian pulse in **Equation. 2.32** we have $\phi(T) = -CT^2/(2T_0^2)$, so that

$$\delta\omega(T) = \frac{CT}{T_0^2}.$$

(Equation 2.35)

Consider the case of negative chirp parameter $C < 0$, then according the **Equation. (2.35)** the instantaneous frequencies in the trailing edge of the pulse $T > 0$ are down-shifted with respect to the carrier frequency, whereas those in the leading edge $T < 0$ are up-shifted. Recall that for normal dispersion the velocity of light tends to decrease with frequency. Then for normal dispersion $\beta_2 > 0$ the down-shifted frequencies in the trailing edge will travel faster than the up-shifted frequencies in the leading pulse edge, allowing the trailing pulse edge to catch up with the leading edge, and the pulse will tend to compress upon propagation. This provides an intuitive explanation of the initial pulse compression for $C = -2$ in **Figure. 2.8**. We shall use the

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notion of instantaneous frequency shift in our treatment of nonlinear optics in fibers in modules 4 and 5.

For additional reading on the material covered in this section see Chapter. 3.2 of Ref. [2].

Some contemporary engineering problems that require a knowledge of the material taught in this module are

- Modeling and design of fibers for communications.
- Modeling of pulse propagation in optical fiber links.
- Pulse propagation in dispersive optical media.

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