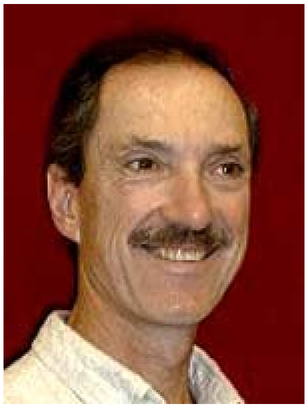
Module 9 – Receivers



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Introduction

In this chapter the detector properties that are important to optical receiver operation are examined. This includes detector frequency response and noise properties. Different receiver amplifiers are discussed and analyzed to investigate their frequency response and noise properties. Receiver detection limits are then examined to determine the minimum detectable power required for different bit error rates (BER). Systems limited by statistics associated with high illumination levels and in the quantum limit are compared. The chapter concludes with a discussion of how other factors such as the signal extinction ratio and timing jitter affect performance.

9.1 Performance of p-i-n photodiodes

One of the most important properties of a detector used in an optical communications system is its ability to respond to modulation of the incident optical beam. The three primary factors that limit the frequency response of a detector are the diffusion time for charge carriers formed outside the depletion region, carrier transit time across the depletion region, and the capacitance set up by the space charge region.

Diffusion time constant

Charge carriers formed outside the space charge region must diffuse to the edge of this region before they can be swept across the space charge region and contribute to the photocurrent.

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This introduces a delay in the photon-current conversion process.

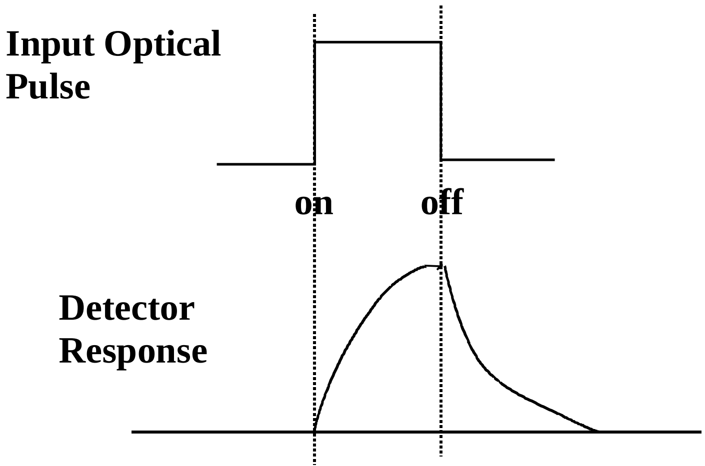

***Figure 9.1** Figure showing the electrical current (lower) in response to a square optical input pulse (upper).*

The detector response function shows an exponential delay:

$$I = I_0 \left[1 - \exp(t / \tau_{diff})\right] \qquad \textbf{\textit{(Equation 9.1)}}$$

The diffusion time constant is given by

$$\tau_{diff} = \frac{d^2}{D_c} \qquad \textbf{\textit{(Equation 9.2)}}$$

Where D_c is the diffusion coefficient for a material like silicon $D_c \sim 3.4 \times 10^{-3}$ m^2/sec and the parameter d is the depth in which absorption occurs. It can be estimated to be the space charge region although it must also include the diffusion distance in addition to the space charge depth.

For example, with d = 5 µm, $\tau_{diff} \sim 7.35$ nsec

Therefore it is important to reduce the diffusion component.

Drift time

This is the time it takes for carriers formed in the space charge region to move through it under the influence of the electric field in this region.

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$$\tau_{drift} = \frac{W}{v_{sat}}$$ (Equation 9.3)

W is the width of the space charge region and v_{sat} is the saturation velocity of charge carriers moving through the space charge region.

The magnitude of the electric field that can be applied across the space charge region is ~ 10^7V/m. With a voltage of this magnitude v_{sat} ~ 10^5m/s in silicon and Drift time τ_{drift} ~ 5×10^{-11}sec/μm.

The carrier velocities are higher in materials such as GaAs and InP due to larger carrier mobility values.

Junction capacitance

In many cases this is the most significant contribution to the frequency response.
The space charge region of the photodiode consists of stationary charges with opposite signs on the two sides of the junction. This results in a capacitance that affects the frequency response of the photodiode receiver circuit.

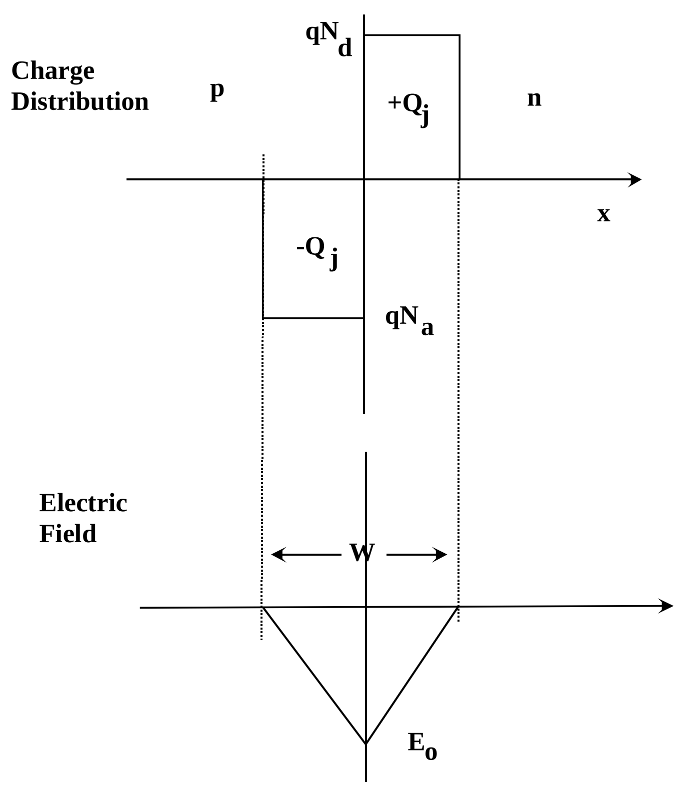

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***Figure 9.2** Figure showing the charge distribution and the corresponding electrical field that builds up across the space charge region of a photodiode.*

The resulting ***junction capacitance*** is

$$C = \frac{\varepsilon_s A}{W} = \left[\frac{\varepsilon_s N_A N_D}{2(N_A + N_D)\cdot(V_o - V_A)} \right]^{1/2} \qquad \textit{(Equation 9.4)}$$

ε_s	= permittivity of the semiconductor
W	= width of the SCR
N_A, N_D	= dopant densities
V_o	= built in potential across the junction
V_A	= applied voltage (negative)

Detector Frequency Response

If the detector is designed to have a large space charge region with a high electric field the diffusion effects can be minimized. The transit time limit to frequency response occurs because it takes a finite time for photogenerated carriers to transit the space charge region. This results in a phase delay between an incident modulated optical signal and the generation of modulated photocurrent.

The analysis starts with the continuity equation applied to photogenerated carriers traversing the i-layer of a p-i-n photodiode with a reverse bias potential V_r. These equations may be stated as:

$$\begin{aligned} \frac{\partial p}{\partial t} &= \frac{-(p - p_o)}{\tau} + G - \frac{1}{q}\frac{\partial J_p}{\partial x} \\ \frac{\partial n}{\partial t} &= \frac{-(n - n_o)}{\tau} + G - \frac{1}{q}\frac{\partial J_n}{\partial x} \end{aligned}, \qquad \textit{(Equation 9.5)}$$

where G is the electron-hole generation rate and τ is the recombination lifetime. The corresponding current density equations are:

$$\begin{aligned} J_p &= q\mu_p pE - qD_p \frac{\partial p}{\partial x} \\ J_n &= q\mu_n nE + qD_n \frac{\partial n}{\partial x} \\ J &= J_n + J_p + \varepsilon_s \frac{\partial E}{\partial t} \end{aligned}, \qquad \textit{(Equation 9.6)}$$

with E the electric field and ε_s the relative permittivity.

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Generally the transit time of charge carriers is much shorter than the recombination lifetime allowing the recombination term to be neglected from the continuity equations. Also if the electric field is large the diffusion term can be neglected.

Consider an incident optical signal of the form:

$$\Phi_{inc} = \Phi_o\left(1+e^{j\omega t}\right). \qquad \textit{(Equation 9.7)}$$

The e-h pair generation rate and carrier densities become:

$$\begin{aligned} G(x,t) &= G_o(x) + G_1(x)e^{j\omega t} \\ n(x,t) &= n_b(x) + n_1(x)e^{j\omega t} \\ p(x,t) &= p_b(x) + p_1(x)e^{j\omega t} \end{aligned} , \qquad \textit{(Equation 9.8)}$$

where the \`b' denotes the dc bias response and the \`1' indicates the response to the modulation amplitude. Using these factors the continuity equations become:

$$\begin{aligned} \frac{\partial J_{n1}}{\partial x} - j\omega\frac{J_{n1}}{v_n} &= -qG_1 \\ \frac{\partial J_{p1}}{\partial x} - j\omega\frac{J_{p1}}{v_p} &= -qG_1 \end{aligned} , \qquad \textit{(Equation 9.9)}$$

where v_n and v_p are the electron and hole saturation velocities and $G_1 = \alpha\Phi_o e^{-\alpha x}$. Applying the boundary conditions $J_{n1}(0)=0;\ J_{p1}(W)=0$, the current density modulation amplitudes become:

$$\begin{aligned} J_{p1}(x) &= -\alpha q\Phi_o\left[\frac{e^{-\alpha x} - e^{-\alpha W + \frac{j\omega(W-x)}{v_p}}}{\left(\alpha - j\omega/v_p\right)}\right] \\ J_{n1}(x) &= -\alpha q\Phi_o\left[\frac{e^{j\omega x/v_n} - e^{-\alpha x}}{\left(\alpha + j\omega/v_p\right)}\right] \end{aligned} . \qquad \textit{(Equation 9.10)}$$

Applying Maxwell's Ampere circuital law to the detector:

$$\nabla\times\bar{H} = \bar{J}_c + \frac{\partial\varepsilon_s\bar{E}}{\partial t}, \qquad \textit{(Equation 9.11)}$$

with a short circuit load causes the E field to be zero. Taking the divergence of both sides leads to: $\nabla\cdot\bar{J} = 0$ and taking the integral in x:

$$J = \frac{1}{W}\int_0^W\left(J_c + \varepsilon_s\frac{\partial E}{\partial t}\right)dx. \qquad \textit{(Equation 9.12)}$$

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In short circuit the applied voltage terms are zero. This results in the frequency dependent current density:

$$J(\omega) = \frac{1}{W}\int_0^W (J_{n1}(x) + J_{p1}(x))dx \,. \qquad \textit{(Equation 9.13)}$$

Substituting for J_{n1} and J_{p1} and carrying out the integration yields:

$$\begin{aligned} J(\omega) &= q\Phi_o \alpha W \left[\frac{e^{-\alpha W} - 1}{\alpha W\left(\alpha W - j\omega\tau_{p,W}\right)} + \frac{e^{-\alpha W}\left(e^{j\omega\tau_{p,W}} - 1\right)}{j\omega\tau_{p,W}\left(\alpha W - j\omega\tau_{p,W}\right)} \right] \\ &+ q\Phi_o \alpha W \left[\frac{1 - e^{j\omega\tau_{n,W}}}{j\omega\tau_{p,W}\left(\alpha W + j\omega\tau_{n,W}\right)} + \frac{1 - e^{-\alpha W}}{\alpha W\left(\alpha W + j\omega\tau_{n,W}\right)} \right] \end{aligned}, \qquad \textit{(Equation 9.14)}$$

where $\tau_{n,W}$ and $\tau_{p,W}$ are the electron and hole transit times across the space charge region with width W.

The current density $J(\omega)$ can be considered to be a current source in the circuit model for the detector. It provides the intrinsic component of the frequency response of the detector to an incoming modulated optical signal. When used as a current source it can be combined with the junction capacitance and other receiver characteristics to determine the overall frequency response of the detector/receiver. The overall frequency response can be written as:

$$J_o(\omega) = J(\omega)H(\omega), \qquad \textit{(Equation 9.15)}$$

where $J_o(\omega)$ is the current across the load and $H(\omega)$ is the transfer function of the detector/receiver circuit.:

$$H(\omega) = \frac{R_{sh}}{A + j\omega\left(B - C\omega^2\right) - D\omega^2} \qquad \textit{(Equation 9.16)}$$

where

$$\begin{aligned} A &= R_s + R_L + R_{sh} \\ B &= R_s R_L C_x + L_s + \left(R_s + R_L\right) R_{sh} C_j + R_L R_{sh} C_x \\ C &= R_s L_s C_x R_{sh} C_j \\ D &= \left(R_s R_L C_x + L_s\right) R_{sh} C_j + R_s L_s C_x + R_{sh} C_x L_s \end{aligned} . \qquad \textit{(Equation 9.17)}$$

C_j and C_x are the junction and external parasitic capacitance, R_{sh} is the shunt resistance of the detector resulting from carriers in the depletion region, R_s is the series resistance resulting from the contact resistance and the resistance of the quasi-neutral regions of the diode, L_s is the parasitic inductance, and R_L is the load resistance of the receiver. Typical values for these parameters are:

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$$R_{sh} = 100 - 500M\Omega$$
$$R_s = 1 - 10\Omega$$
$$R_L = 50 - 75\Omega$$
$$C_j = 50 - 100\,fF$$
$$C_x = 10 - 20\,fF$$
$$L_s = 50 - 100\,pH$$

(Equation 9.18)

The magnitude of the frequency response $J_o(\omega)$ will generally decrease with increasing modulation frequency.

9.2 Detector Circuit Model

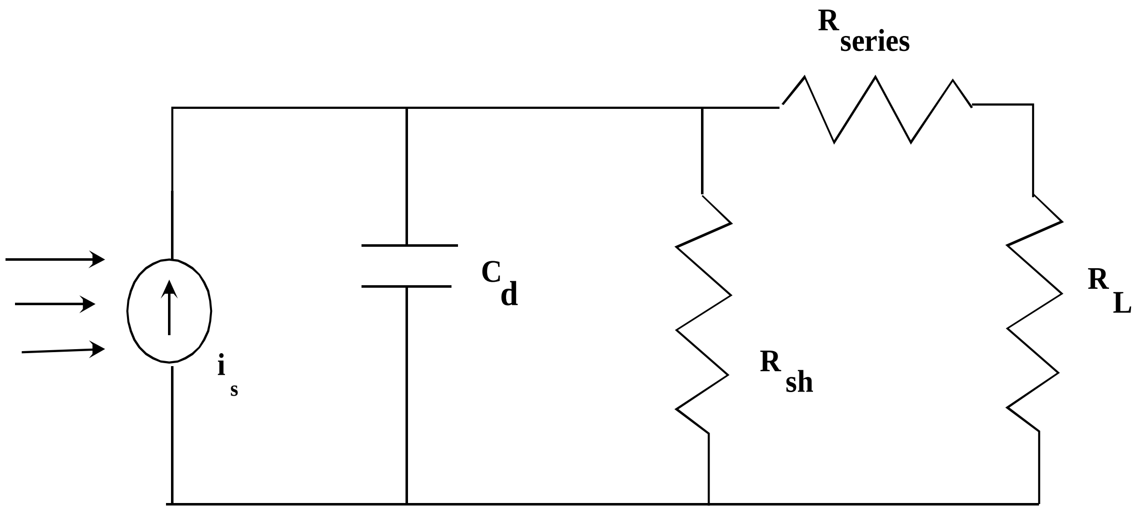

***Figure 9.3** The schematic of a circuit model for a photodiode with a load attached. R_L is the load resistance; C_d is the detector capacitance; R_{sh} is the shunt resistance; R_{series} is the series resistance; i_s is the optically generated current of the detector.*

In this type of circuit, typically the internal series resistance is relatively low and the internal shunt resistance is high. Therefore the detector can be modeled in simplified form as a current source in parallel with the junction capacitance.

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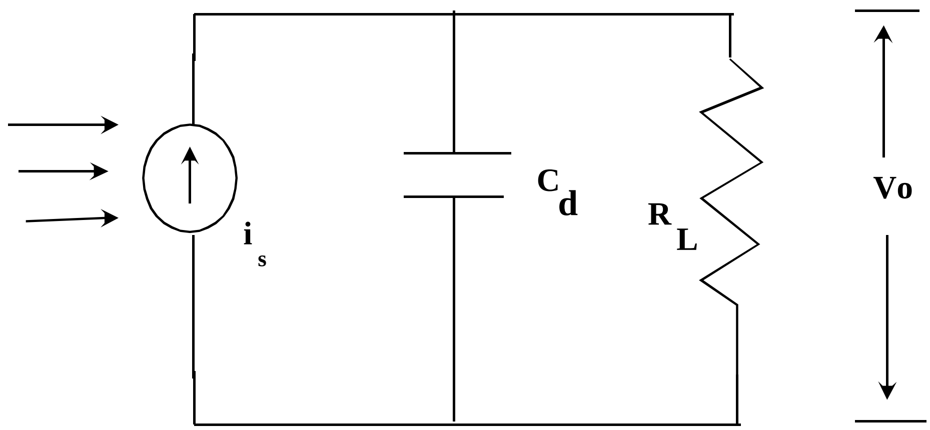

Figure 9.4 *Simplified circuit model for a photodiode. The series and shunt resistance are negligible in comparison to other circuit parameters.*

This circuit is representative of a High Impedance Front End Receiver. It allows high voltage output since R_L can be made large. The frequency response can be found from:

$$v_o = i_s \frac{RZ_C}{R + Z_C} \qquad \textit{(Equation 9.19)}$$

$$Z_C = \frac{1}{j\omega C} \qquad \textit{(Equation 9.20)}$$

$$v_o = i_s \frac{R}{1 + R/Z_C} = i_s \left[\frac{R}{1 + j\omega RC} \right] \qquad \textit{(Equation 9.21)}$$

$$v = i_s \frac{R}{\left[1 + (\omega RC)^2\right]} (1 - j\omega RC) \qquad \textit{(Equation 9.22)}$$

Therefore the 3-dB (50%) frequency is:

$$B_{3dB} = \frac{1}{2\pi RC} \qquad \textit{(Equation 9.23)}$$

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Relation between B_{3dB} and rise time

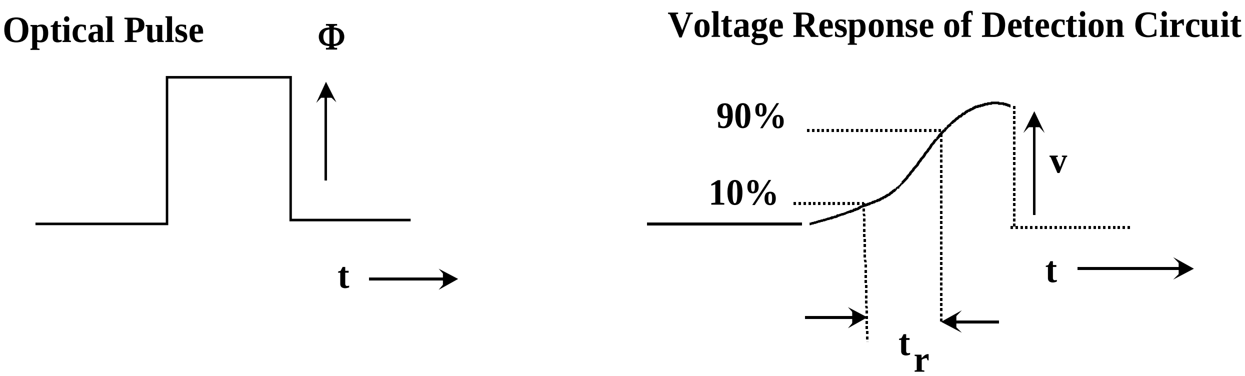

Figure 9.5 ***Figure of an incoming optical pulse and the rise time of a voltage response of a detector to the optical pulse.***

The RC circuit of the detector circuit can be formed into a differential equation. Solving the equation for the *rise time* (i.e. the time for the voltage to rise from 10% V_{max} to 90%V_{max} level) gives:

$$t_r = 2.19 R_L C_d \qquad \textit{(Equation 9.24)}$$

As shown earlier the 3-dB bandwidth is related to the RC factor by

$$B_{3dB} = \frac{1}{2\pi R_L C_d}. \qquad \textit{(Equation 9.25)}$$

Using $R_L C_d$ as a common factor we obtain a relation between t_r and B_{3dB}

$$R_L C_d = \frac{t_r}{2.19} = \frac{1}{2\pi B_{3dB}} \qquad \textit{(Equation 9.26)}$$

$$\therefore t_r = \frac{2.19}{2\pi B_{3dB}} = \frac{0.345}{B_{3dB}}. \qquad \textit{(Equation 9.27)}$$

Transimpedance Amplifier

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The difficulty with the high impedance front end is the large RC time constant. One way to eliminate this problem is to use a transimpedance amplifier. In this system the output is connected to the input. Although this seems rather strange at first it has many benefits. A diagram of this configuration is shown below.

Transimpedance Detector Amplifier

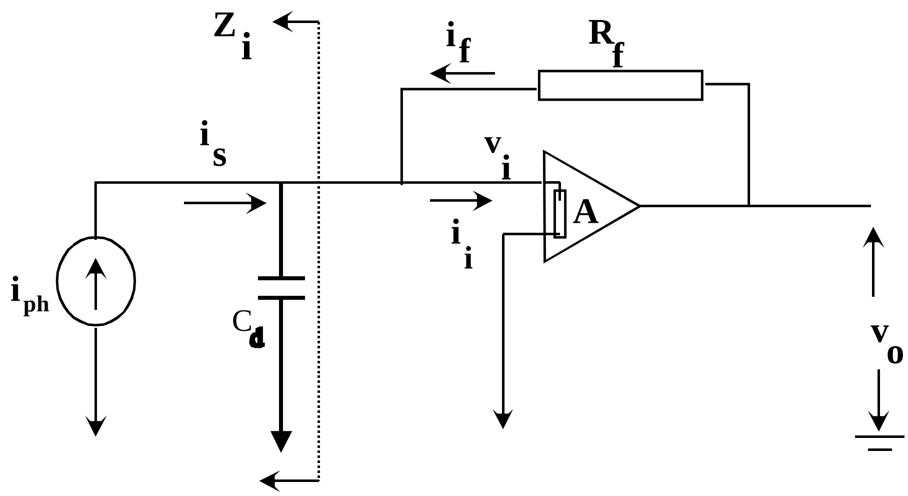

Figure 9.6 *Schematic of an optical detector (photodiode) and a transimpedance amplifier.*

Assumptions:

For an ideal Op Amp:

$i_I \approx 0$

$i_s \approx i_f$

$v_o = A\, v_i$

$$v_o - v_i = i_f R_f \qquad \textit{(Equation 9.28)}$$

$$v_o\left[1 - \frac{1}{A}\right] = i_s R_f \qquad \textit{(Equation 9.29)}$$

$\therefore$ the output voltage of the transimpedance amplifier is determined by the feedback resistance.

For large gain A: $v_o \approx i_s R_f$; therefore the transimpedance amplifier output voltage is the same as the high impedance termination case with $R_F = R_L$. However, there will be a difference in bandwidth as shown below.

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Input impedance Z_I: (determines the limit on BW and is the impedance of the detector).

The input impedance is defined as:

$$Z_i = \frac{\partial v_i}{\partial i_s} \quad \textit{(Equation 9.30)}$$

$$v_o = Av_i = \frac{i_s R_f}{1 - \frac{1}{A}} \quad \textit{(Equation 9.31)}$$

$$v_i = \frac{1}{A}\left[\frac{i_s R_f}{1 - \frac{1}{A}}\right] \quad \textit{(Equation 9.32)}$$

$$\therefore Z_i = \frac{1}{A}\frac{R_f}{\left(1 - \frac{1}{A}\right)} = \frac{R_f}{A-1} \quad \textit{(Equation 9.33)}$$

$\therefore$ The input impedance can be reduced by increasing the gain of the amplifier.

This can be used to increase the bandwidth of the receiver. However the ***circuit noise*** is still a function of the feedback resistance R_f. This aspect will be discussed later in the section on detector noise.

9.3 Receiver Sensitivity

Bit Error Rate - BER

A measure of a good receiver is to have the same performance with the lowest level of incident optical power.

Performance can be measured as a low bit error rate (BER). BER is the probability of an incorrect identification of a bit by the decision circuit of a receiver.

A BER of 3×10^{-7} implies that 3 bits will be in error for every 10 million received bits. For communications systems the BER is less than 10^{-9}. As the data rates increase the BER must be significantly lower than this value.

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Receiver Sensitivity is the minimum average received optical power $\overline{P}_{\min}$ required to achieve a fixed BER.

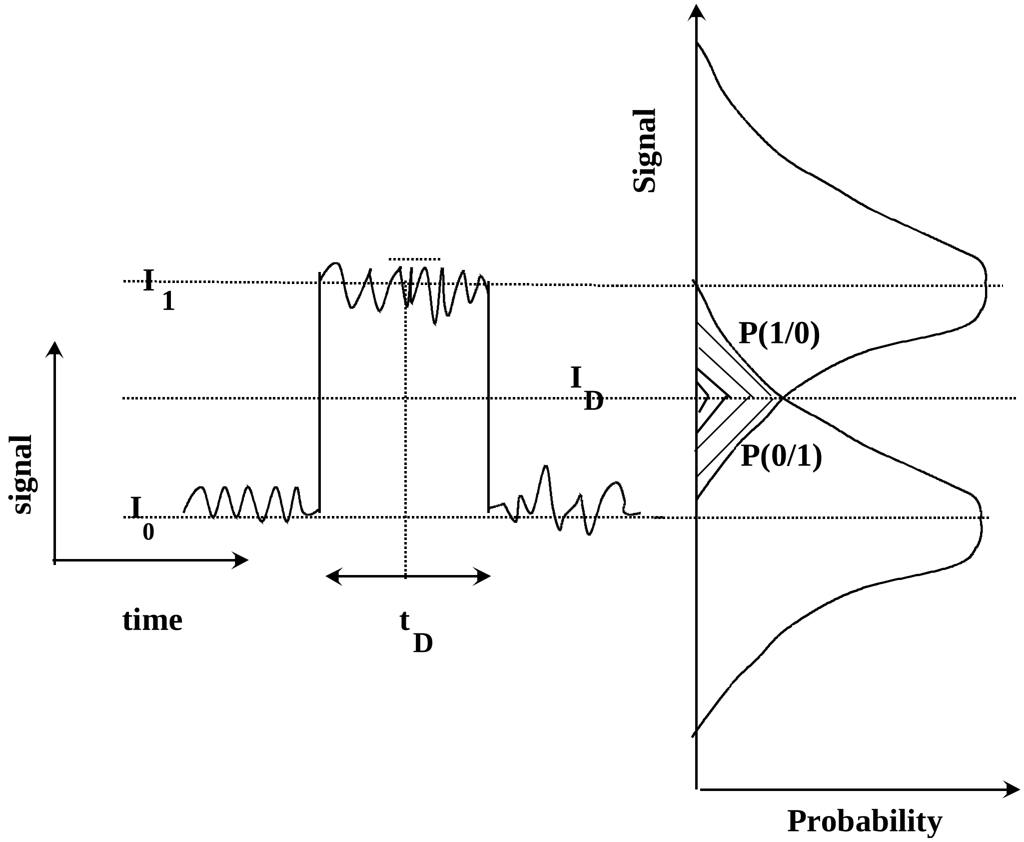

Figure 9.7 *Figure showing an input `1' and `0' current signal received by a detector and an exaggerated distribution for the received signals. The fluctuation is expressed as a probability function. When enough photons are received the distribution will look like a Gaussian distribution as predicted by the central limit theorem in probability theory.*

If $I > I_D$ a 1 bit is recorded.
If $I < I_D$ a 0 bit is registered.

An error occurs for a 1 bit if $I < I_D$ and similarly an error for a zero bit takes place when $I > I_D$.

These errors can be included in the error probability as

$$BER = p(1)P(0/1) + p(0)P(1/0) \qquad \textit{(Equation 9.34)}$$

where p(1) and p(0) are the probabilities of receiving a 1 and 0 bit respectively; P(0/1) is the probability of deciding 0 when a 1 is sent and P(1/0) is the probability of deciding 1 when a 0 is sent.

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Assuming that 1s and 0s are equally likely, p(1) = p(0) = 0.5

$$BER = 0.5\left[P(0/1) + P(1/0)\right].$$ ***(Equation 9.35)***

Both *shot* and *thermal noise* can be modeled with *Gaussian statistics* (for large numbers of electrons).

The total *variance in the current* for I_1 or I_0 can be modeled as

$$\sigma_{1,0}^2 = \left\langle i_{s,1,0}^2 \right\rangle + \left\langle i_{T,1,0}^2 \right\rangle$$ ***(Equation 9.36)***

The *average noise currents* squared were previously described for shot and thermal noise.
Since the *average current I_p* is different for the 1 and 0 levels the shot noise and the variance will be different.
Consequently in general σ_1 and σ_0 will also differ.

The conditional probabilities based on Gaussian statistics are:

$$P(0/1) = \frac{1}{\sigma_1\sqrt{2\pi}}\int_{-\infty}^{I_D}\exp\left(-\frac{(I-I_1)^2}{2\sigma_1^2}\right)dI = \frac{1}{2}erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right)$$ ***(Equation 9.37)***

$$P(1/0) = \frac{1}{\sigma_0\sqrt{2\pi}}\int_{I_D}^{\infty}\exp\left(-\frac{(I-I_0)^2}{2\sigma_0^2}\right)dI = \frac{1}{2}erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)$$ ***(Equation 9.38)***

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-y^2)dy$$ ***(Equation 9.39)***

Gaussian statistics are appropriate with a large number of photons/bit.
This results in a BER of:

$$BER = \frac{1}{4}\left[erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)\right].$$ ***(Equation 9.40)***

I_D is chosen to minimize BER. This occurs when

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$$\left(I_1 - I_D\right)/\sigma_1 = \left(I_D - I_0\right)/\sigma_0 \equiv Q$$ ***(Equation 9.41)***

and when the noise of the 1 and 0 levels are not equal

$$I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}.$$ ***(Equation 9.42)***

When $\sigma_1 = \sigma_0, \rightarrow\ I_D = (I_1 + I_0)/2$
or the *decision threshold current I_D* lies midway between I_0 and I_1.

In this case:

$$BER = \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp\left(-Q^2/2\right)}{Q\sqrt{2\pi}}$$.***(Equation 9.43)***

The approximation is good for Q > 3. A BER ≈ 10^{-9} occurs with Q ~ 6.
A general expression for Q that is true even when $\sigma_1 \neq \sigma_o$

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$ ***(Equation 9.44)***

Minimum Received Optical Power

Since the currents I_1 and I_0 are related to the optical power through the responsivity, it is possible to determine the minimum optical power to obtain a fixed BER. Minimum optical power will be used as a metric for evaluating system performance.

For the case with $I_0 = 0$, $P_0 = 0$. The detector current for the 1 bit is

$$I_1 = MRP_1 = 2MR\overline{P}_{\min},$$ ***(Equation 9.45)***

where $\overline{P}_{\min}$ is the *average received minimum power* with

$$\overline{\mathrm{P}}_{\min} = (\mathrm{P}_1 + \mathrm{P}_0)/2.$$ ***(Equation 9.46)***

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Noise terms include both thermal and shot noise for the 1 bit but only thermal noise for the 0 bit:

$$\sigma_1 = \left(\sigma_s^2 + \sigma_T^2\right)^{1/2}; \sigma_0 = \sigma_T \, . \qquad \textit{(Equation 9.47)}$$

Therefore for an APD:

$$Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2MR\overline{P}_{\min}}{\left(\left\langle i_s^2 \right\rangle + \left\langle i_T^2 \right\rangle\right)^{1/2} + \frac{< i_T^2 >^{1/2}}{M}} \, . \qquad \textit{(Equation 9.48)}$$

Solving for $\overline{P}_{\min}$:

$$\overline{P}_{\min} = \frac{Q}{\Re}\left(qF_A QB_e + \frac{\left\langle i_T^2 \right\rangle^{1/2}}{M}\right) \qquad \textit{(Equation 9.49)}$$

For a PIN detector:

$$\overline{P}_{\min} = \frac{Q}{\Re}\left(qQB_e + \left\langle i_T^2 \right\rangle^{1/2}\right) \qquad \textit{(Equation 9.50)}$$

Quantum Limit for Photodetection

When only a few photons arrive per data bit the assumption that was used for Gaussian statistics is no longer valid. In this case it is still possible to formulate the detection process however a different probability distribution function must be used that represents the quanta arrival of photons.

In this case Poisson statistics should be applied.

For the analysis let:
N_p = average number of photons in each `1’ bit;
k = # of *e-h* hole pairs being formed in each bit to produce electrical current;

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Then the probability of forming k *e-h* pairs is given by:

$$P(k) = \exp(-N_p) N_p^k / k! \qquad \textit{(Equation 9.51)}$$

If it is assumed that there is no power in the `0' bit, then from the conditional probabilities defined earlier:

P(1/0) = 0; probability of getting a `1' when a 0 was transmitted;
P(0/1) = exp(-N_p); probability of getting a `0' when a `1' was transmitted.

Using the previous definition for BER:

$$BER = 0.5[P(0/1) + P(1/0)] \qquad \textit{(Equation 9.52)}$$

$$\therefore BER = \exp(-N_p)/2\,. \qquad \textit{(Equation 9.53)}$$

The minimum received optical power in this case is

$$\overline{P}_{\min} = N_p h\nu B/2\,. \qquad \textit{(Equation 9.54)}$$

At 1.55 μm hν = 0.8 eV, and the $\overline{P}_{\min}$ = 13 nW (-48.9 dBm @ 10 Gb/s).

In most receivers however N_p is typically ~ 1000 which is considerably higher than the quantum limit.

Extinction Ratio

In many cases the power in the 0-bit will not be 0.
The extinction ratio – can be defined as

$$r = P_o / P_1\,. \qquad \textit{(Equation 9.55)}$$

In this case

$$Q = \left(\frac{1-r}{1+r}\right)\frac{2R\overline{P}_{\min}}{\sigma_1 + \sigma_0} \qquad \textit{(Equation 9.56)}$$

Minimum required optical power becomes:

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$$\overline{P}_{min} = \left(\frac{1+r}{1-r}\right) \cdot \frac{(\sigma_1 + \sigma_0)}{2} \cdot \left(\frac{Q}{R}\right) \qquad \textit{(Equation 9.57)}$$

Effect of Intensity Noise

Most optical sources will exhibit some form of output power fluctuation. This fluctuation is another source of system noise and must be considered in the SNR. Assuming that all noise sources are independent the total current noise power term can be represented as:

$$\langle i_N^2 \rangle = \langle i_s^2 \rangle + \langle i_T^2 \rangle + \langle i_I^2 \rangle . \qquad \textit{(Equation 9.58)}$$

The *laser power fluctuation* can be expressed as:

$$\langle i_I \rangle = R \left\langle \left(\Delta P_{inc}^2 \right) \right\rangle^{1/2} , \qquad \textit{(Equation 9.59)}$$

With ***source noise*** the ***Q parameter*** becomes (with $I_0 = 0$ and $\sigma_0 = \sigma_T$):

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{2R\overline{P}_{min}}{\left(\langle \sigma_T^2 \rangle + \langle \sigma_s^2 \rangle + \langle \sigma_I^2 \rangle \right)^{1/2} + \sigma_T} \qquad \textit{(Equation 9.60)}$$

with

$$\sigma_s = \left(4qR\overline{P}_{rec} B_e \right)^{.5} ; \sigma_I = \langle i_I \rangle \qquad \textit{(Equation 9.61)}$$

Effects of Timing Jitter

The previous receiver sensitivity analysis is based on the assumption that the signal is sampled at the peak of the voltage pulse.

Timing decisions are typically determined by a clock-recovery circuit.

Since the input to this circuit is noisy the sampling time fluctuates from bit to bit.

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This fluctuation is referred to as timing jitter and will degrade the SNR.

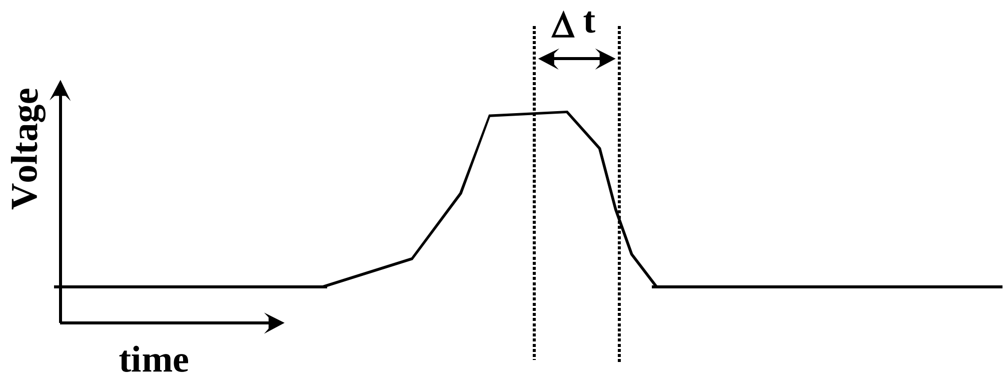

Figure 9.8 *Diagram representing an actual voltage signal for a `1' bit as a function of time. If the voltage signal is sampled at a point away from the peak level a `1' bit can be mistaken for a zero since it has a voltage below the threshold for a `1' bit.*

Since Δt is a *random variable* the *change in the sampled voltage level* is also a *random variable*. This reduces the receiver performance.

The SNR can be improved at the expense of higher source power.

Effect on System Performance

Assume a *pin* detector/receiver that is limited by thermal noise with (extinction ratio) r = 0, and $I_0 = 0$.

$$Q = \frac{I_1 - \langle \Delta i_j \rangle}{\left(\sigma_T^2 + \sigma_j^2\right)^{.5} + \sigma_T} \qquad \textit{(Equation 9.62)}$$

The new quality factor (Q) will increase the minimum power required to achieve a predetermined BER. For a *pin* photodiode the impact of jitter on $\bar{P}_{\min}$ can be determined from the relation:

$$\bar{P}_{\min} = \frac{Q}{\Re}\left(qQB_e + \langle i_T^2 \rangle^{1/2}\right). \qquad \textit{(Equation 9.63)}$$

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